Nonsingular vacuum cosmologies with a variable cosmological term

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We present nonsingular cosmological models with a variable cosmological term described by the second-rank symmetric tensor Λ_μ^ν evolving from $\Lambda\delta_\mu^\nu$ to $\lambda\delta_\mu^\nu$ with $\lambda < \Lambda$. All Λ_μ^ν dominated cosmologies belong to Lemaître type models for an anisotropic perfect fluid. The expansion starts from a nonsingular nonsimultaneous de Sitter bang, with Λ on the scale responsible for the earliest accelerated expansion, which is followed by an anisotropic Kasner type stage. For a certain class of observers these models can be also identified as Kantowski-Sachs models with regular R regions. For Kantowski-Sachs observers the cosmological evolution starts from horizons with a highly anisotropic "null bang" where the volume of the spatial section vanishes. We study in detail the spherically symmetric case and consider the general features of cosmologies with planar and pseudospherical symmetries. Nonsingular Λ_μ^ν dominated cosmologies are Bianchi type I in the planar case and hyperbolic analogs of the Kantowski-Sachs models in the pseudospherical case. At late times all models approach a de Sitter asymptotic with small λ.

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I. INTRODUCTION

Anything which contributes to the energy density of vacuum $\rho_{\rm vac}$ and satisfies the equation of state $p = -\rho$ (we adopt $c = 1$ for simplicity), is associated with the Einstein cosmological term by [1,2] (for reviews see [3–5])

$$T_\mu^\nu = \rho_{\rm vac}\delta_\mu^\nu = (8\pi G)^{-1}\Lambda\delta_\mu^\nu \qquad (1)$$

The cosmological constant Λ must be constant by the conservation equation $\nabla_\nu T_\mu^\nu = 0$ that follows from the contracted Bianchi identities $\nabla_\nu G_\mu^\nu = 0$ and the Einstein equations $G_\mu^\nu = -8\pi G T_\mu^\nu$.

Developments in particle and quantum field theories as well as inflationary scenarios and observational cosmology compellingly suggest that the cosmological constant Λ has to be a dynamical quantity.

In inflationary models in which an accelerated expansion due to Λ is based on generic properties of the de Sitter vacuum (1), independently of where Λ comes from [6], various mechanisms have been proposed relating the cosmological term $\Lambda\delta_\mu^\nu$ to matter sources (for reviews see [7,8]). A huge cosmological constant or vacuum density at the earliest stage of the Universe evolution ($\rho_{\rm vac} \sim 10^{77}$ g·cm^{-3} for the Grand Unification scale $E_{\rm GUT} \sim 10^{15}$ GeV) provides an accelerated expansion needed to guarantee the survival of the Universe to its present size and density [6] as well as to explain its observed homogeneity and isotropy and solve other puzzles of the standard Big Bang cosmology (for a review and a list of puzzles see [7,8]). On the other hand, cosmological observations require the present value of the vacuum density to provide a substantial contribution (about 70 per cent) to the total density in the Universe, $\rho \sim 10^{-30}$ g·cm^{-3} [9].

Most of the models for Λ dynamics considered in the literature are of "cancelling type" (for reviews see [4,5,10]), involving fields (most frequently scalar fields

non-minimally coupled to gravity [11,12]) which develop a negative energy density growing with time to ultimately cancel a pre-existing (driving inflation) positive value of Λ. This approach typically requires the gravitational constant G to be time- [11] or scale- [13] dependent (the question of confronting G-variable models with observations is addressed in Ref. [14]). Models with a dynamical Λ-term depending binomially on the scalar field are proposed in the context of scalar-tensor theories with matter described as a perfect fluid with a barotropic equation of state [15].

Another wide class of models are phenomenological FRW (homogeneous and isotropic) cosmologies with an effective stress-energy tensor (SET) describing both the Λ-term and non-vacuum matter as perfect fluids (for a review see [5]). In Ref. [5], a number of new exact singular FRW solutions are presented for the case $\Lambda \propto t^{-l}$ and $k = 0$ (spatially flat isotropic models), with $\Lambda < 0$ for odd values of l; several numerical models are built for $k = 1$ (a closed universe) with Λ varying as a function of the scale factor a, and for the case of an open universe ($k = -1$) with Λ varying as a function of the Hubble parameter H. The paper [16] presents a nonsingular isotropic cosmology with a phenomenological ansatz postulating Λ decay from an initial de Sitter state which ideologically resembles the old nonsingular FRW model with the de Sitter vacuum as an initial state [6].

In Ref. [17], plane-symmetric Bianchi type I models are considered with a perfect fluid source including dust, radiation and time-dependent Λ. The evolution starts from an initial singularity followed by a highly anisotropic stage which involves a big negative value of Λ at the beginning in the case of the critical density [17]. In Ref. [18], isotropic but inhomogeneous spherically symmetric cosmologies are considered with a material fluid, a heat flux, a Q matter (related to a self-interacting scalar field) and a time-dependent cosmological constant

Λ; late-time izotropization depends on the time evolution of an effective adiabatic index for the material fluid [18].

In this paper we present nonsingular cosmological models with a variable cosmological term described by the second-rank symmetric tensor Λ_μ^ν [19] in which a constant Λ associated with vacuum density by (1), becomes a tensor component $\Lambda_t^t = (8\pi G)T_t^t$ associated explicitly with the density component of the perfect fluid SET $T_\mu^\nu = (8\pi G)^{-1}\Lambda_\mu^\nu$ [19] whose vacuum properties follow from its symmetry and whose variability follows from the Bianchi identities [20]. This approach allows us to specify possible types of cosmological models with variable vacuum density consistent with the Bianchi identities.

Let us emphasize that our approach is phenomenological in essence.

In quantum field theory, the symmetry of the vacuum expectation value of the SET $\langle T_{\mu\nu} \rangle$ does not always coincide with the symmetry of the background space-time, since quantum field theory in curved space-time does not contain a unique specification of the quantum state of a system [21–23]. Even in Minkowski space-time there are vacuum states (e.g., the well-known Rindler vacuum) which are non-invariant under the full Poincaré group. In the case of de Sitter space the renormalized expectation value of the vacuum SET $\langle T_{\mu\nu} \rangle$ for a scalar field with an arbitrary mass m and curvature coupling ξ is proved to have a fixed point attractor behavior at late times [23] approaching, depending on m and ξ, either the Bunch-Davies de Sitter-invariant vacuum [24] or, for the massless minimally coupled case ($m = \xi = 0$), the de Sitter-invariant Allen-Folacci vacuum [25]. The case $m = \xi = 0$ is peculiar (see, e.g., [26]), since the de Sitter-invariant two-point function is infrared divergent, and the vacuum states free of this divergence are $O(4)$-invariant Fock vacua introduced by Allen [27]. The vacuum energy density in the $O(4)$-invariant case is not the same (larger) than in the de Sitter-invariant case [26].

The problems of specifying vacuum states in quantum field theory are beyond the scope of this paper.

We here address the question: what can be the further cosmological evolution for cases when the vacuum density (no matter where it came from) had once contributed to a SET of the form (1).

Phenomenologically, the SET (1) is defined as a vacuum SET due to its maximally symmetric form invariant under any coordinate transformations which makes impossible to single out a preferred comoving reference frame [1,28]. As a result, an observer moving through a medium with a SET of structure (1) cannot in principle measure his velocity with respect to it, which allows one to classify it as a vacuum [1,29].

It is clear that if one wants to make variable a vacuum density defined in this way, one cannot retain the full invariance of the vacuum SET. The invariance can, however, be partial, valid in a certain Lorentzian subspace of the space-time manifold. In this case, the full symmetry is reduced but the invariance is still present for an observer moving along a certain direction in space.

This approach has been proposed in Ref. [20] (for a review see [30,31]), where a spherically symmetric vacuum has been introduced as defined by the algebraic structure of its SET such that

$$T_t^t = T_r^r, \qquad T_\theta^\theta = T_\phi^\phi \qquad (2)$$

We adopt the metric signature $(+ - - -)$. The energy density is given by $\rho = T_t^t$, the radial pressure is $p_r = -T_r^r$ and the transversal pressure is $p_\perp = -T_\theta^\theta = -T_\phi^\phi$.

The vacuum properties of (2) follow from its invariance under boosts in the radial direction which results in the absence of a preferred comoving reference frame and makes impossible for an observer to measure the radial component of his velocity [20].

The vacuum tensor of this kind was first introduced as a source term for an exact spherically symmetric solution describing a black hole whose singularity is replaced with a de Sitter vacuum core [20,32]. Later it was shown that the existence of the class of solutions to the Einstein equations with the source term of the form (2), asymptotically de Sitter at the center, is distinguished by the dominant energy condition and by the requirements of regularity of density and finiteness of the ADM mass [36]. A SET from this class smoothly connects two de Sitter vacua — at the center and at infinity, and corresponds to an extension of the Einstein cosmological term $\Lambda\delta_\mu^\nu$ to the second-rank symmetric tensor with the algebraic structure (2) [19],

$$\Lambda_\mu^\nu = 8\pi G T_\mu^\nu \qquad (3)$$

evolving from $\Lambda\delta_\mu^\nu$ as $r \to 0$ to $\lambda\delta_\mu^\nu$ as $r \to \infty$, with $\lambda < \Lambda$. The Bianchi identities result in the conservation equation $\Lambda_{\nu;\mu}^\mu = 0$ which gives the r-dependent equation of state [19]

$$p_r^\Lambda = -\rho^\Lambda, \qquad p_\perp^\Lambda = -\rho^\Lambda - \frac{r}{2}\frac{d\rho^\Lambda}{dr} \qquad (4)$$

where $\rho^\Lambda(r) = (8\pi G)^{-1}\Lambda_t^t$, $p_r^\Lambda = -(8\pi G)^{-1}\Lambda_r^r$ and $p_\perp^\Lambda = -(8\pi G)^{-1}\Lambda_\theta^\theta = -(8\pi G)^{-1}\Lambda_\phi^\phi$. The global structure of the Λ_μ^ν geometry contains black and white holes whose singularities are replaced with regular cores, asymptotically de Sitter as $r \to 0$. A regular core of a white hole considered in the cosmological coordinates (R, τ), models the early stages of an expanding universe dominated by Λ_μ^ν; the evolution starts from a nonsingular non-simultaneous de Sitter bang (a bang is defined by $r(R, \tau) = 0$ [37]) followed by a Kasner-type stage of anisotropic expansion at which most of the mass is produced [38].

In this paper we study Λ_μ^ν-dominated cosmologies in a more general cosmological context. Namely, if there is a spatial direction distinguished by the symmetry of the source, we can choose a class of coordinate frames where this direction is parametrized by a certain coordinate u. Then our choice of the vacuum SET is only restricted by the requirement that it should be invariant under coordinate transformations in the (u, t) subspace, or, in other words, under boosts in the distinguished two-dimensional Lorentzian subspace $\mathbb{M}^2(u, t)$ of the four-dimensional space-time manifold. It is then natural to

assume that the two-dimensional subspaces orthogonal to $\mathbb{M}^2$ are maximally symmetric, i.e., spaces of constant curvature. The latter can be spheres $\mathbb{S}^2$, planes $\mathbb{R}^2$ or Lobachevsky planes $\mathbb{L}^2$, attached to each point of $\mathbb{M}^2$ with a certain scale factor which depends on a point in $\mathbb{M}^2$, i.e., in general on both coordinates u and t.

The paper is organized as follows. Sec. II presents the basic equations and exact solutions with a variable cosmological term (3) for the cases of spherical, planar and pseudospherical symmetries. Sec. III is devoted to a systematic study of spherically symmetric models. Conclusions are summarized in Sec. IV.

II. Λ_μ^ν DOMINATED GEOMETRIES

A. Basic relations

We consider 4-spaces with the structure

$$\mathbb{M}^4 = \mathbb{M}^2 \times \mathbb{V}^2, \quad \mathbb{V}^2 = \begin{cases} \mathbb{S}^2, & K = 1; \\ \mathbb{R}^2, & K = 0; \\ \mathbb{L}^2, & K = -1. \end{cases} \quad (5)$$

Here $\mathbb{M}^2$ is the Lorentzian 2-subspace parametrized by the coordinates (u, t); $\mathbb{V}^2$ is a 2-surface with spherical ($K = 1$), planar ($K = 0$) or pseudospherical ($K = -1$) symmetries (ϕ and θ in (2) are any coordinates parametrizing $\mathbb{V}^2$). In the cosmological context, all three symmetries can exist on equal rights. (The index K should not be confused with $k = 0, \pm 1$, the spatial curvature index of the FRW models.)

An arbitrary static metric for the source term (2) and the space-time structure (5) is given by

$$ds^2 = A(u)dt^2 - \frac{du^2}{A(u)} - r^2(u)d\Omega_K^2, \quad (6)$$

where $d\Omega_K^2$ is the squared linear element on a unit sphere ($K = 1$), plane ($K = 0$) or Lobachevsky plane ($K = -1$).

The choice of the "radial" coordinate u such that $g_{tt}g_{uu} = -1$ is not only one of the widely used coordinate gauges: it is especially convenient when considering Killing horizons which correspond to zeros of the function $A(u)$. The reason is that in a close neighborhood of a horizon the coordinate u varies (up to a positive constant factor) like manifestly well-behaved Kruskal-like coordinates used for an analytic extension of the metric (6). Therefore one can jointly describe in terms of u the metric on both sides of a horizon. In addition, with the same coordinate, horizons correspond to regular points in geodesic equations [39].

It should be emphasized that the static form chosen for the metric (6) is not a restriction. As soon as we postulate the space-time structure (5) and the SET structure (2), the generalized Birkhoff theorem [40,41] guarantees the existence of a coordinate frame where the metric has the form (6). The only restriction is the assumption that, in the space-time domain under consideration, the metric coefficient $r^2(u, t)$ is not constant and has a non-null

gradient: $\text{sign}(g^{\mu\nu}r_{,\mu}r_{,\nu}) = \pm 1$. If it is $+1$, we are dealing with a T (cosmological) region, and -1 corresponds to an R (static) region.

Indeed, according to (6), $g^{\mu\nu}r_{,\mu}r_{,\nu} = -A(dr/du)^2$. The space-time regions where $A > 0$ are static, the u coordinate being spatial, and are called R regions. In T regions, where $A < 0$, the coordinates u and t interchange their roles: u becomes timelike and t spacelike. In the spherically symmetric case, as is well known, the metric in a T region describes a Kantowski-Sachs (KS) type homogeneous anisotropic cosmological model [42,43], representing a special case of T-models [44], which are, in general, inhomogeneous. A spatial section of a KS model has the structure $\mathbb{R} \times \mathbb{S}^2$, a 3-dimensional cylinder with different time-dependent scale factors in the spherical and longitudinal directions.

In the case of planar symmetry ($K = 0$), in a T region we obtain anisotropic models with planar spatial sections, i.e., Bianchi type I models, more precisely, their plane-symmetric subset where two of the three scale factors coincide.

In the pseudospherical case ($K = -1$), spatial sections in a T-region have the structure $\mathbb{R} \times \mathbb{L}^2$, similar to KS models but with spheres replaced by Lobachevsky planes. We will call such models HKS (hyperbolic Kantowski-Sachs) models.

In regions where $A(u) < 0$, it is convenient to change the notation: $t \mapsto x$ and $-A(u) \mapsto b^2(u)$, remembering that u is now a temporal coordinate. The metric is then rewritten as

$$ds^2 = \frac{1}{b^2(u)}du^2 - b^2(u)dx^2 - r^2(u)d\Omega_k^2, \quad (7)$$

which describes an anisotropic cosmological model with two time-dependent scale factors $b(u)$ and $r(u)$ and the lapse function $1/b^2(u)$. A transition to synchronous time τ is performed using the integral

$$\tau(u) = \int \frac{du}{b(u)}. \quad (8)$$

A horizon $u = h$ of order n is a zero of the same order of the function $A(u)$. For the metric (7), the horizon $u = h$ is a coordinate singularity where the metric coefficient g_{xx} vanishes, so that coordinate surfaces (e.g., spheres in case $K = 1$) with the same finite scale factor $r(h)$ stick to one another. By Eq. (8), this happens at finite cosmological time τ for a simple (first-order) horizon and in an infinitely remote past or future ($\tau \to \pm\infty$) for higher-order horizons.

On the other hand, if a T region is located at large r, the anisotropic cosmology can isotropize at late times τ under the condition $b(\tau) \propto r(\tau)$. One can notice that for $K = \pm 1$ the isotropization can be only local. Indeed, for $K = 1$ the spatial topology is cylindrical, and the directions along and across the coordinate spheres are non-equivalent. For $K = -1$ the spatial topology is flat, but the global geometry is different along and across the Lobachevsky planes.

Two independent components of the Einstein equations and the conservation equation for Λ_μ^ν give the dynamical equations

$$2Ar''/r = \Lambda_u^u - \Lambda_t^t = 0, \tag{9}$$

$$\frac{1}{r^2}\left(-K + Ar'^2 + A'rr'\right) = -\Lambda_t^t = -8\pi G\rho, \tag{10}$$

$$r\rho' + 2(\rho + p_\perp) = 0, \tag{11}$$

where the prime denotes d/du.

Eq. (9) leads to $r'' = 0$. Solutions with $r = $ const can be of certain interest, but not in the cosmological context since they cannot describe an expanding universe with isotropization at late times. Let us note that in case $r = $ const the generalized Birkhoff theorem does not work and, in addition to solutions with the metric (6), there exist wave solutions [40,41]. Here we do not consider such solutions.

Now without loss of generality we can put $u \equiv r$ and consider the remaining unknowns as functions of r. We are left with two equations (10) and (11) for three unknowns $A(r)$, $\rho(r)$, $p_\perp(r)$. The set of equations becomes determined if we postulate an equation of state connecting ρ and $p_\perp$ or specify the density profile $\rho(r)$.

Eq. (10) can be integrated giving

$$A(r) = K - \frac{2GM(r)}{r}, \quad M(r) = 4\pi\int_d^r \rho(x)x^2 dx. \tag{12}$$

with an arbitrary constant d. This is a solution in quadratures for any given $\rho(r)$, $p_\perp$ is then determined from (11).

In the particular case of the equation of state $p_\perp = -\rho$, which leads to the usual cosmological constant, $\lambda = 8\pi G\rho = $ const, the integration (12) gives

$$A(r) = K - \frac{2Gm}{r} - \frac{1}{3}\lambda r^2 \tag{13}$$

which is the extension of Schwarzschild-de Sitter geometry (described by the Kottler-Trefftz solution [45]) to the cases $K = 0, -1$; m is an integration constant ($m = 0$ when $d = 0$). It is interpreted as the mass in case $K = +1$, $\lambda = 0$ (the Schwarzschild solution).

B. De Sitter space-time

This is the maximally symmetric solution to the Einstein equations with the cosmological constant $\Lambda > 0$. It is the special case $m = 0$ of the solution (6), (13), so that the metric is

$$ds^2 = A(r)dt^2 - \frac{dr^2}{A(r)} - r^2 d\Omega_k^2, \tag{14}$$

$A(r) = K - H^2 r^2$, where $H^{-1} = \sqrt{3/\Lambda}$ is the curvature radius of this constant-curvature manifold. Different values of K correspond to the description of the same spacetime in different coordinate frames. Let us briefly discuss the properties of the metric (14) since the textbook descriptions (see, e.g., [21,46]) discuss its static, spherically

symmetric form (which is the $K = 1$ case in our notation) and its FRW forms, while we also need the de Sitter metric in KS and HKS forms (7) to specify the asymptotic behavior of our models.

In case $K = 0$, a static region is absent ($A < 0$), and the metric (7) transforms into the FRW form [46]

$$ds^2 = d\tau^2 - e^{2H\tau}(dx^2 + dy^2 + dz^2), \tag{15}$$

The expansion begins at $\tau \to -\infty$, i.e., in the infinitely remote past of cosmological observers.

In the spherically symmetric case, in the nonstatic region $Hr > 1$ the KS metric for de Sitter space takes the form

$$\begin{aligned}ds^2 &= (H^2r^2 - 1)^{-1}dr^2 - (H^2r^2 - 1)\,dx^2 - r^2 d\Omega_1^2\\ &= d\tau^2 - \sinh^2(H\tau)\,dx^2 - H^{-2}\cosh^2(H\tau)d\Omega_1^2.\end{aligned} \tag{16}$$

The expansion in this model begins at $\tau = 0$ from a highly anisotropic apparently singular state with $g_{xx} = 0$ and becomes locally isotropic (as noticed in Sect. IIA) and exponential (with the Hubble constant H) at large values of τ.

In case $K = -1$, the HKS metric (7) in terms of τ reads

$$ds^2 = d\tau^2 - \cosh^2(H\tau)\,dx^2 - H^{-2}\sinh^2(H\tau)d\Omega_{-1}^2. \tag{17}$$

The expansion begins with an anisotropic apparent singularity at $\tau = 0$, where the Lobachevsky planes are drawn to points. The further expansion, as in other cases, ultimately becomes isotropic and exponential.

Apparent singularities at the start of expansion are in all three cases produced by the choice of coordinate frames since the de Sitter geometry is globally regular and maximally symmetric. All 4-points of the de Sitter manifold (including those which seem to be singular in particular representations of the metric) are equivalent to each other.

Let us note that, comparing the KS and HKS forms of the de Sitter metric with its FRW forms, we see that the topology of spatial sections of the same de Sitter 4-geometry depends on the choice of the coordinate frame. Indeed, the spatial topology is $\mathbb{R}^3$ for an open and $\mathbb{S}^3$ for a closed FRW model, $\mathbb{R} \times \mathbb{S}^2$ for the KS representation (16) and $\mathbb{R}^3$ for the HKS form.

So far we discussed expanding models with the de Sitter metric. Their contracting counterparts are easily obtained by time reversal ($\tau \to -\tau$).

Fig. 1 shows a plot of $A(r)$ and the Carter-Penrose global structure diagram for de Sitter space-time. The metrics (16), (17) describe either the contracting region T_- for a contracting Universe ($\tau \in \mathbb{R}_-$), or T_+, for an expanding Universe ($\tau \in \mathbb{R}_+$). (R regions admit both expansion and contraction.)

C. Cosmologies with a variable cosmological term

Consider now the general case of the solutions (6), (12) with the cosmological term Λ_μ^ν of the algebraic structure

FIG. 1. De Sitter geometry.

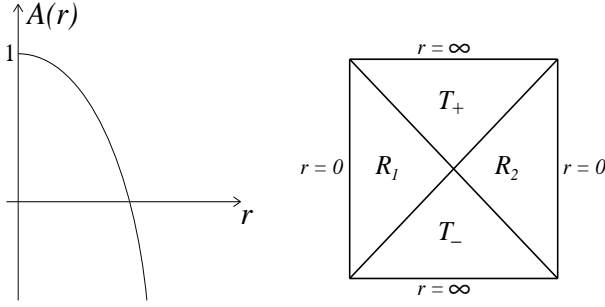FIG. 2. The metric function $A(r)$ for the de Sitter-Schwarzschild geometry. The mass parameter is normalized to a critical mass.

(2). The solutions of the class to be considered satisfy the dominant energy condition ($\rho \geq 0$, $|p_i| \leq \rho$, $i = 1, 2, 3$), are regular at $r \to 0$ and tend to the de Sitter metric with a finite value of the density ρ as $r \to \infty$ [36]. The derivative of the density is $\rho' = -2(\rho + p_\perp)/r$ by (11), and $\rho' < 0$ follows from the dominant energy condition, so that $\rho(r) = 8\pi G \Lambda_t^t$ is a monotonically decreasing function of r. This guarantees the existence of cosmological solutions evolving from a large value of the cosmological constant at small r to its small value at large r.

It can be verified by using the Kretchmann scalar $\mathcal{R}^2 = R_{iklm} R^{iklm}$, where R_{iklm} is the Riemann curvature tensor, that, for the class of metrics under consideration, regularity at $r = 0$ is only achieved if $A(r)$ behaves at small r as $k - A_2 r^2 + o(r^2)$, where A_2 is a non-negative constant. This means that any solution with Λ_μ^ν, regular at $r = 0$, is asymptotically de Sitter at small r (with $A_2 = H^2$ in the above notations) not only in the spherically symmetric case [36], but also in the cases of planar and pseudospherical symmetries.

According to (14), the spherical model ($K = 1$) contains a regular R region near $r = 0$, while the planar and pseudospherical models ($K = 0, -1$) contain regular T regions near $r = 0$.

On the other hand, Eq. (10) implies that on a horizon, where $A = 0$,

$$rA' = K - 8\pi G \rho r^2. \qquad (18)$$

For $K = 0, -1$ this means that $A' < 0$ at any horizon, i.e., it must be a simple horizon leading, as r increases, from R to T region. Since, as we have seen, regularity at $r = 0$ requires that the region near $r = 0$ is a T region (with a de Sitter behavior), we have to conclude that all models considered here with $K = 0$ and $K = -1$ have no horizon, comprise T regions only and are purely cosmological. They describe anisotropic evolution between states with large and small values of cosmological constant, but effects connected with the existence of horizons are absent.

In case $K = 0$ the model is classified as a Bianchi type I model and in case $K = -1$ as a hyperbolic Kantowski-Sachs model.

Spherically symmetric systems are much more diverse and complicated, and we discuss them in more detail in the next section.
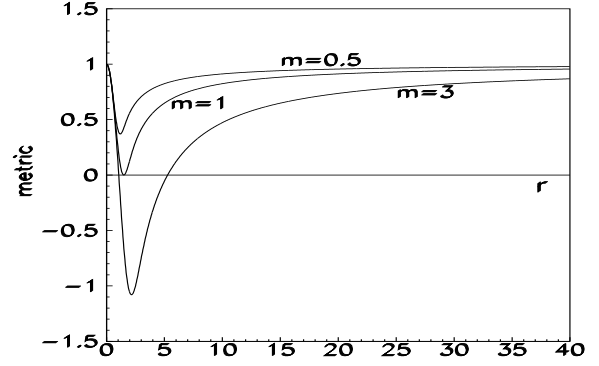
III. SPHERICALLY SYMMETRIC MODELS

A. One- and two-lambda configurations

In the spherically symmetric case, $K = 1$, the metric has the form

$$ds^2 = A(r)dt^2 - \frac{dr^2}{A(r)} - r^2(d\theta^2 + \sin^2\theta \, d\phi^2). \qquad (19)$$

Requiring regularity at the center $r = 0$, we put $d = 0$ in (12), and $M(r)$ is, as usual, interpreted as the mass inside a sphere of radius r. Near the center the solution approaches the de Sitter metric with $\Lambda = 8\pi G\rho(0)$ [36]. The function $A(r)$ is given by

$$A(r) = 1 - \frac{2GM(r)}{r}, \quad M(r) = 4\pi \int_0^r \rho(x)x^2 dx. \qquad (20)$$

If we require asymptotic flatness, then the monotonically decreasing function $\rho(r)$ should vanish as $r \to \infty$ quicker than r^{-3}, and the total gravitating mass (ADM mass) $m = M(\infty)$ is finite [36]. The de Sitter-Schwarzschild geometry, asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$, describes a vacuum nonsigular black hole for masses $m \geq m_{crit}$ where m_{crit} is a critical value, and a particlelike structure without horizons for $m < m_{crit}$ in the Minkowski background [20,34] (see Fig. 2).

The global structure of de Sitter-Schwarzschild spacetime for $m \geq m_{crit}$ contains an infinite set of vacuum nonsingular black and white holes whose singularities are replaced by regular cores with de Sitter vacuum near $r = 0$ (see Fig. 3).

The regular core R of a white hole T_+ models a nonsingular start of cosmological evolution which we consider in the next section.

This geometry is easily extended to the case of a nonzero cosmological constant $\lambda < \Lambda$ at infinity [47]. The solution is described by the metric function [47]

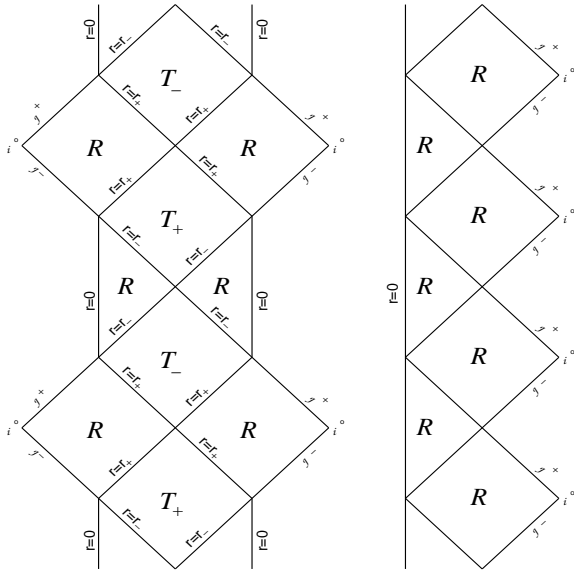$$A(R) = 1 - \frac{2GM(r)}{r} - \frac{\lambda}{3}r^2 \qquad (21)$$

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FIG. 3. Global structure of de Sitter-Schwarzschild space-time with two horizons (left) and one double horizon (right).
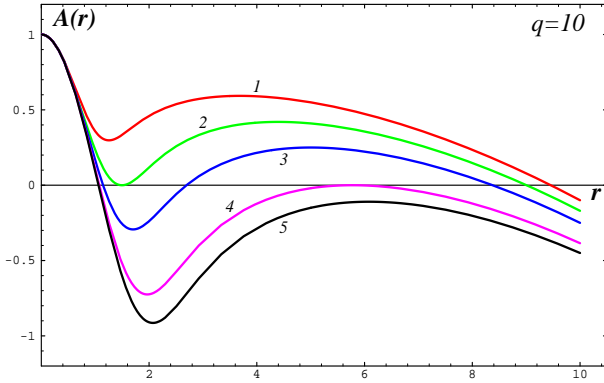

FIG. 4. Configurations described by Λ_μ^ν.

with $\rho \to \rho_\infty = \lambda/(8\pi G)$ as $r \to \infty$. For a particular density profile of this kind [20],

$$\rho(r) = \rho_0 \exp(-r^3/r_1^3) + \rho_\infty \qquad (22)$$

(where ρ_0, r_1 and ρ_∞ are constants), $A(r)$ is shown in Fig. 4, where $q = \sqrt{\Lambda/\lambda}$ [47]. The cosmological term Λ_μ^ν smoothly connects two de Sitter vacua with different values of the cosmological constant. The geometry describes five types of globally regular configurations with qualitatively different behavior of $A(r)$ (see Fig. 4 [47,48]).

The global structure of space-time contains in this case cosmological T regions which are asymptotically de Sitter as $r \to \infty$. The particular structure depends on the number and nature of horizons, which in turn depend on the values of the parameters in (22).

According to Eqs. (10) and (11), the transversal pressure $p_\perp$ can be expressed in terms of the function $A(r)$:

$$8\pi G p_\perp = \frac{1}{2}A'' + \frac{A'}{r}. \qquad (23)$$

At an extremum of $A(r)$, $A' = 0$, hence, if $p_\perp > 0$, this extremum is a minimum, and this minimum of A is unique in the domain where $p_\perp > 0$ (otherwise there would be a maximum between two minima). Assuming that $p_\perp$ becomes negative only on distinguished length scales l_i related to particular symmetry breakings with a de Sitter-like (false vacuum) behavior involved, we can fix the maximum number of horizons. One scale l_1 is related to the de Sitter core near the center. If there is no other symmetry breaking scale, we have an asymptotically flat configuration where $A(r)$ is positive at both small and large r and has only one minimum, hence no more than two zeros (horizons) [36]. If there is a de Sitter asymptotic with small λ, this gives another scale l_2. Then there can be again only one domain $p_\perp > 0$, but now A has different signs at $r \to 0$ and $r \to \infty$, hence the single minimum of A leads to at most 3 horizons: an internal horizon r_-, a black hole horizon r_+, and a cosmological horizon r_{++}.

Let us specify the possible types of cosmological models corresponding to globally regular configurations described by the cosmological term Λ_μ^ν.

All Λ_μ^ν-dominated cosmologies belong to the Lemaître class of anisotropic fluid models. The cosmological expansion starts with a nonsingular non-simultaneous bang which we consider in detail in the next section.

For a certain class of observers in T regions (specified in Sec. IIIC), Λ_μ^ν-dominated cosmologies can also be identified as Kantowski-Sachs models with a regular R region.

In cases 1 and 5 the global structure of space-time is the same as for the de Sitter geometry, but now the dynamics is governed by a variable cosmological term. The difference between the models of types 1 and 5 is related to the particular dynamics. In case 1 the main dynamics occurs in the R region, and in case 5 in the T region.

In cases 2, 3 and 4 in Fig. 4 the global structure is more complex and is depicted in Figs. 5–7 [49]. Particles moving in these geometries from the center to the asymptotically de Sitter T regions have to cross two horizons in cases 2 and 4, three horizons in case 3, and in cases 3 and 4 they have to cross an intermediate T region.

In the case of three horizons the global structure is shown in Fig. 5. (For a static observer in the R region between the black hole horizon r_+ and the cosmological horizon r_{++}, it corresponds to a nonsingular cosmological black hole [47]).

The global structure of space-time for the case when the black hole horizon r_+ coincides with the internal horizon r_- is shown in Fig. 6.

When the black hole horizon coincides with the cosmological horizon, the global structure is shown in Fig. 7.

It is also possible that $A(r)$ has a triple horizon; this gives again the de Sitter space-time structure.

In all these models, the cosmological evolution for a Kantowski-Sachs observer in the T region starts from a horizon, i.e., a null surface with vanishing volume of the spatial section. We therefore call it a *null bang*. In the
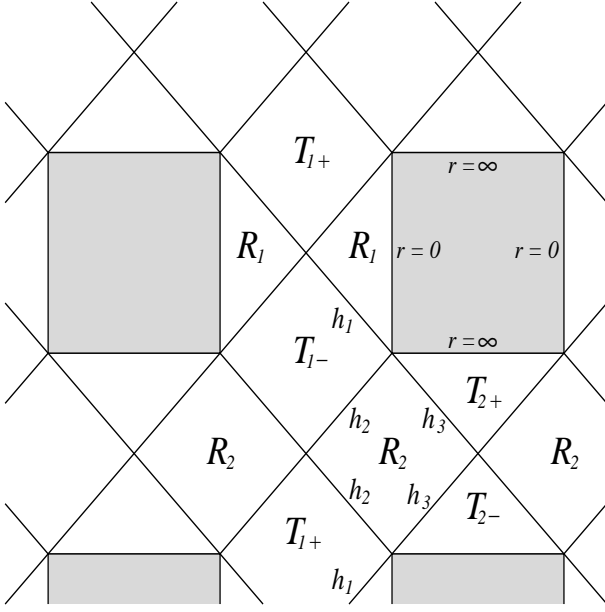
FIG. 5. Global structure of the Λ_μ^ν geometry with three horizons.
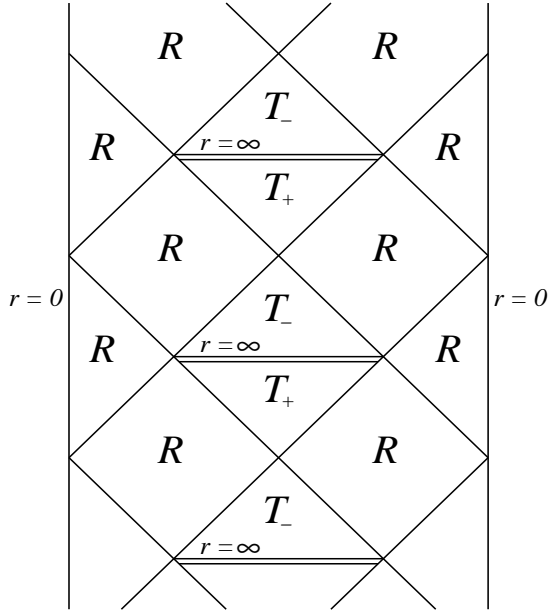

FIG. 6. Global structure of the Λ_μ^ν geometry with the double horizon $r_- = r_+$.

case of a double horizon a null bang occurs in the infinite past of an observer. In cases 1 and 5, as well as in the de Sitter case, the evolution starts from a simple horizon and at finite time from the point of view of a Kantowski-Sachs observer.

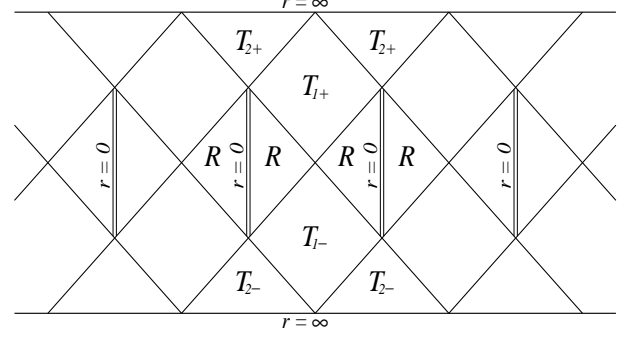The cosmological properties of these models are discussed in detail in the next sections.

FIG. 7. Global structure of the Λ_μ^ν geometry with the double horizon $r_+ = r_{++}$.

B. Lemaître cosmologies with a nonsingular, nonsimultaneous bang

To present the Λ_μ^ν geometry as a Lemaître-type spherically symmetric model, we connect an expanding reference frame with a certain congruence of radial geodesics and consider this frame as comoving to our cosmological model. The procedure is similar to what Lemaître has done for the Schwarzschild metric [29]. The algebraic structure (2) of Λ_μ^ν implies that a reference frame connected with any radial motion will automatically be comoving to our vacuum matter.

A transition to the geodesic coordinates (R, τ), where τ is the proper time along a geodesic and the radial coordinate R is the congruence parameter, different for different geodesics, can be described in a general form. A radial timelike geodesic in the metric (19) satisfies the equations

$$\left(\frac{dr}{d\tau}\right)^2 = E^2 - A(r), \qquad \frac{dt}{d\tau} = \frac{E}{A(r)}, \qquad (24)$$

where the constant E is connected with the initial velocity of a particle moving along this particular geodesic at a given value of r. Relating the reference frame to geodesics with a certain fixed value of E, we can perform a transition from the coordinates (r, t) to the coordinates (R, τ) with the following transformation:

$$\partial_\tau r = \pm\sqrt{E^2 - A}, \qquad \partial_R r = \sqrt{E^2 - A}, \qquad (25)$$

$$\partial_\tau t = \frac{E}{A}, \qquad\qquad \partial_R t = \pm\frac{E^2 - A}{AE}, \qquad (26)$$

where the plus and minus signs refer to growing and falling $r(\tau)$ (expanding and contracting models), respectively. The resulting metric has the form

$$ds^2 = d\tau^2 - \frac{E^2 - A(r)}{E^2}dR^2 - r^2(R, \tau)d\Omega_1^2, \qquad (27)$$

where $r(R, \tau)$ should be determined from Eqs. (25). For expanding models, since $\partial_\tau r = \partial_R r$, we see that r is a function of $R + \tau$. (In this construction we have used the freedom of re-parametrization $R \mapsto \tilde{R}(R)$.)

Such a model for de Sitter-Schwarzschild geometry has been considered in Ref. [38] for radial geodesics with

7

$E = 1$ in the present notations. According to (24), the condition $E = 1$ means that the velocity of particles realizing our reference frame is zero at $r = 0$. As a result, the center $r = 0$, where the expansion starts, corresponds to the surface $R + \tau = -\infty$ in the coordinates (R, τ). Different points of the bang surface $r(R, \tau) = 0$ [37] (which is time-like in Λ^ν_μ geometry) start at different instants of synchronous time τ. In the limit $r \to 0$ the metric transforms to the de Sitter metric

$$ds^2 = d\tau^2 - e^{2H\tau}(dx^2 + x^2 d\Omega_1^2) \qquad (28)$$

with $H = \sqrt{\Lambda/3}$ and $x = H^{-1} e^{HR}$ and describes a non-singular nonsimultaneous de Sitter bang [38], followed by a Kasner-type anisotropic stage, with contraction in the radial direction and expansion in the tangential direction, at which most of the Universe mass is produced; the metric at the intermediate stage has the form [38]

$$ds^2 = d\tau^2 - (\tau+R)^{-2/3}F(R)dR^2 - B(\tau+R)^{4/3}d\Omega_1^2 \quad (29)$$

where $F(R)$ is a smooth regular function and B is a constant related to the model parameters.

Similar models can be constructed in the general case with one, two or three Killing horizons. In all Λ^ν_μ geometries the regular world line $r = 0$ is timelike (see Figs. 3, 5), therefore different particles that realize our reference frame leave or cross this world line at different instants of synchronous time τ. The start of the expansion is therefore the same in the general case, i.e., a nonsingular nonsimultaneous de Sitter bang at $r = 0$, occuring at $R + \tau \to -\infty$ for $E = 1$ and at finite $R + \tau$ if $E > 1$, when the initial velocity of the geodesic reference frame is nonzero. This inflationary stage is followed by a highly anisotropic stage of evolution. Expansion in the tangential direction ($\partial_\tau r > 0$) is accompanied by contraction in the radial direction ($\partial_\tau |g_{RR}| < 0$) as long as $dA/dr > 0$, as is easily seen from (27). At large values of $R + \tau$, when the metric achieves another de Sitter asymptotic corresponding to the cosmological constant λ, the model becomes isotropic and is again described by the metric (28), but now with the Hubble parameter $H = \sqrt{\lambda/3}$.

C. Kantowski-Sachs cosmologies with a null bang

The asymptotically de Sitter regions of the above two-lambda configurations represent, as any spherically symmetric T regions, KS cosmological models. The metric has the form (7) with the temporal coordinate $u = r$ and $b^2(u) = |A(r)|$:

$$ds^2 = \frac{1}{b^2(r)}dr^2 - b^2(r)dx^2 - r^2 d\Omega_1^2 \qquad (30)$$

The KS evolution at late times corresponds to the de Sitter large r asymptotic (16). In all cases the KS evolution starts with a null bang from a horizon. This initial state with a finite value of r is, from the viewpoint of comoving observers in the model (30), a highly anisotropic, purely coordinate singularity where the spatial sections, having
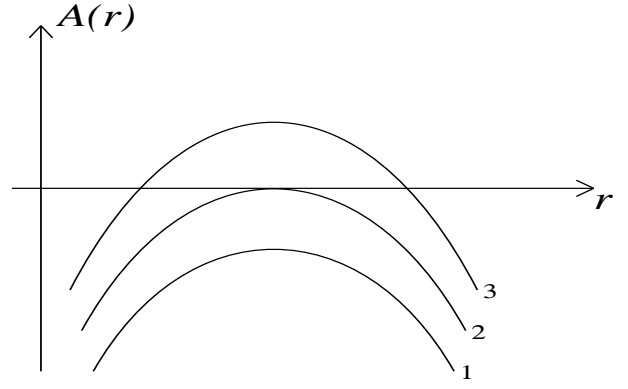

FIG. 8. The function $A(r)$ for the Schwarzschild-De Sitter geometry

the topology of a 3-dimensional cylinder, are squeesed along the "longitudinal" direction x due to vanishing $b(r)$. The 4-geometry is, however, globally regular.

In cases 1, 2, 3 and 5 according to Fig. 4, this initial state occurs at finite proper time $\tau = \int dr/b(r)$. Thus our KS models behave qualitatively as the de Sitter model (see Sec. 2.2), but with quite different prehistories. In case 4, with a double horizon connecting two T regions, the null bang occurs in the remote past, $\tau \to -\infty$. The same is true for the case of a triple horizon.

Let us show that observers who follow other geodesics cross this past horizon at finite instants of their proper times. Indeed, the geodesic equations in any space-time with the metric (6) have the following integral:

$$(dr/d\tau)^2 + k_1 A + L^2 A/r^2 = E^2 \qquad (31)$$

where E and L are the constants of motion associated with the particle energy and angular momentum, while $k_1 = 1$ and $k_1 = 0$ for timelike and null geodesics, respectively; the affine parameter τ has the meaning of proper time along the geodesic in case $k_1 = 1$. At a horizon $r = h$, $A(r)$ vanishes, and in case $E > 0$ one has $dr/d\tau \neq 0$ for all geodesics, whence it follows that $|\tau| < \infty$, irrespective of the order of the horizon. The time lines in the T region which are trajectories of the KS observers, evidently correspond to $k_1 = 1$, $L = 0$, $E = 0$, so that for these lines we return to the relation (8), leading to infinite τ at horizons of order two and higher.

Therefore even "slow" observers whose world lines coincide with these time lines receive information from their infinitely remote past, coming with particles and photons which have crossed the horizon.

The reasoning related to Eq. (31) is quite general, and in other KS cosmologies, starting with a null bang, the prehistory is also observable.

One can take as a simple example the Schwarzschild-de Sitter metric (19), (13) with $K = 1$, $m > 0$ and $\lambda > 0$. The metric function $A(r) = 1 - 2Gm/r - (\lambda/3)r^2$ is plotted in Fig. 8.

Three cases should be distinguished:
(i) $3Gm > 1/\sqrt{\lambda}$, no horizon (curve 1). It is a global KS model with a highly anisotropic singularity $r = 0$ in the past and a de Sitter future asymptotic.

(ii) $3Gm = 1/\sqrt{\lambda}$, one double horizon $h = 3Gm$, separating two T regions (curve 2). The region $r > h$ is similar to the large r region in case 4 of the Λ^{ν}_{μ} geometry: a null bang in the infinitely remote past.

(iii) $3Gm < 1/\sqrt{\lambda}$, two horizons h_1 and h_2 (curve 3). The region $r < h_1$ is quite similar to the Schwarzschild black hole interior, while the region $r > h_2$ has the same qualitative features as the large r region in cases 1, 2, 3 and 5 of the Λ^{ν}_{μ} geometry (Fig. 4), and the null bang takes place at finite proper time.

As before, observers in the KS regions receive signals from the pre-historical inner regions. Thus in Schwarzschild-de Sitter space-time the past anisotropic singularity is visible to all observers.

One more case of interest for comparison (though with a different structure of the SET) is that of a scalar field minimally coupled to gravity, with the Lagrangian

$$L_{\varphi} = g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - 2V(\varphi), \tag{32}$$

where $V(\varphi)$ is a potential. This field with various potentials is applied to various cosmological and local problems. If φ is constant, the system reduces to vacuum in general relativity with the cosmological constant given by the corresponding value of V=const. As shown in Ref. [51], in the case of static, spherically symmetric configurations, a variable scalar field adds nothing to the list of possible global space-time structures with constant φ, whatever is the choice of the function $V(\varphi)$ (not necessarily positive-definite). In particular, there can be no more than two horizons; the only possible double horizon separates two T regions. If there is a black hole horizon (i.e., a horizon $r = h$ separating a T region at $r < h$ from an R region at $r > h$), there is always a singularity at $r = 0$. The system with any small r behaviour has a de Sitter asymptotic at infinity if $\varphi \to \varphi_0$ such that $V(\varphi_0) > 0$ at large r.

A common feature with the Λ^{ν}_{μ} geometry is that scalar field configurations which have a regular center are always asymptotically de Sitter at $r = 0$, but in the scalar case they can have either no horizon, or one cosmological horizon.

Therefore the above discussion of regular Λ^{ν}_{μ} geometries (cases 1 and 5 according to Fig. 4) is applicable to regular scalar field configurations with a cosmological horizon. The discussion of the Schwarzschild-de Sitter geometry is applicable to the class of systems with scalar fields, singular at $r = 0$.

A qualitative picture similar to cases 1 and 5 according to Fig. 4 is also observed if a single scalar field is replaced with a scalar triplet forming a global monopole configuration [52]: in a certain range of the gravitational field strength parameter, the monopole has a de Sitter core, separated by a cosmological horizon from an outer KS region with a large r asymptotic, characterized by a zero or small cosmological constant [53].

IV. SUMMARY AND DISCUSSION

Let us briefly summarize our results.

We have presented regular Λ^{ν}_{μ} dominated perfect fluid models in which the evolution starts from the de Sitter vacuum $\Lambda\delta^{\nu}_{\mu}$ with Λ on the scale of symmetry breaking (which can be a Grand Unification or SUSY scale) and ultimately approaches another de Sitter vacuum $\lambda\delta^{\nu}_{\mu}$ with $\lambda < \Lambda$.

These models belong to the Lemaître class of anisotropic fluid models. The cosmological expansion starts from a nonsingular non-simultaneous de Sitter bang, followed by a Kasner type anisotropic stage and approaches isotropic FRW expansion at late times.

Exact solutions giving rise to Λ^{ν}_{μ} dominated models are presented for three symmetries (spherical, planar and pseudospherical) of spatial 2-surfaces. All planar and pseudospherical regular models are anisotropic T models (without R regions) with isotropization at late times. The planar case is classified as the Bianchi I type and the pseudospherical case can be called the hyperbolic Kantowski-Sachs class.

In the spherically symmetric case the models contain regular R regions and can be identified as Kantowski-Sachs models with regular R regions.

A remarkable feature of these models is that, for a Kantowski-Sachs observer, the evolution starts with a "null bang", from a null surface which seems singular to comoving observers but which is perfectly regular in the 4-dimensional Λ^{ν}_{μ} geometry. Information about the pre-null bang history is available to Kantowski-Sachs observers.

The KS and Lemaître type cosmologies presented here are built on the basis of the same space-times but differ in the choice of reference frames. This is an example of the evident circumstance that the observational properties of space-time in general relativity depend on the observers' motion, in other words, on the choice of a reference frame. Another, well-known example is the de Sitter metric which, being written in different reference frames, gives rise to all three types of FRW isotropic models. Two anisotropic representations of the de Sitter geometry are given here in Eqs. (16) and (17).

We are currently working on detailed cosmological models consistent with all observational constraints. Our preliminary results testify that Λ^{ν}_{μ} cosmologies are able to provide a smooth decay of vacuum density by more than hundred orders of magnitude during the cosmological evolution.

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